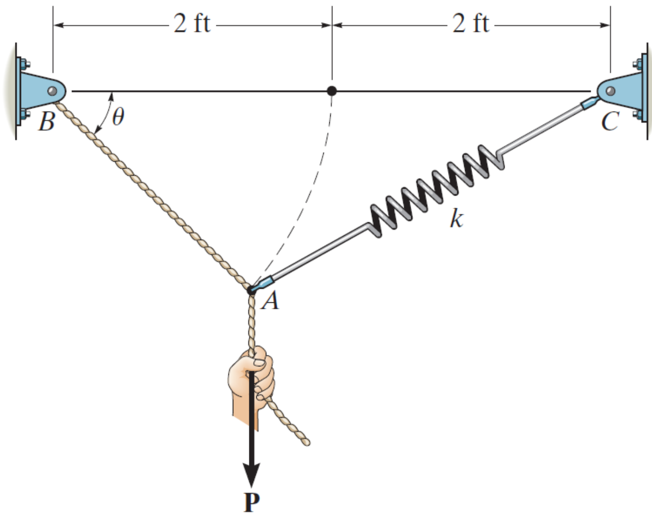
Problem 3-20

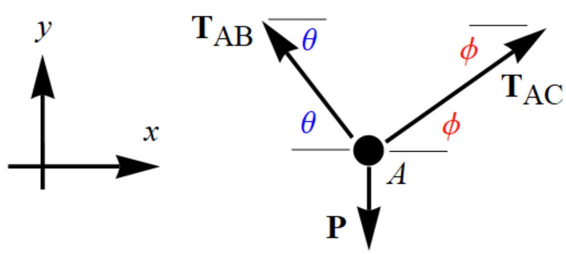
A vertical force $P = 10$ lb is applied to the ends of the 2-ft cord AB and spring AC. If the spring has an unstretched length of 2 ft, determine the angle θ for equilibrium. Take $k = 15$ lb/ft.



Probs. 3–20/21

Solution

Draw a free-body diagram for the point at A.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

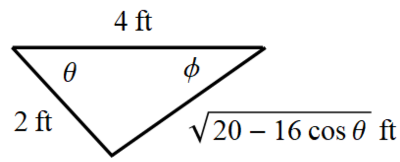
$$\sum F_x = 0: \qquad T_{AC}\cos\phi - T_{AB}\cos\theta = 0$$

$$\sum F_y = 0: \qquad T_{AC}\sin\phi + T_{AB}\sin\theta - P = 0$$

Since $P = 10$ lb and $T_{AC} = k\Delta x_{AC} = (15 \text{ lb/ft})\left[\sqrt{2^2 + 4^2 - 2(2)(4)\cos\theta} \text{ ft} - 2 \text{ ft}\right]$, the system of equations reduces to

$$15\left(\sqrt{20 - 16\cos\theta} - 2\right)\cos\phi - T_{AB}\cos\theta = 0 \tag{1}$$

$$15\left(\sqrt{20 - 16\cos\theta} - 2\right)\sin\phi + T_{AB}\sin\theta - 10 = 0. \tag{2}$$



Use the law of sines to determine $\sin\phi$.

$$\frac{\sqrt{20-16\cos\theta}\text{ ft}}{\sin\theta}=\frac{2\text{ ft}}{\sin\phi}\quad\rightarrow\quad\sin\phi=\frac{2\sin\theta}{\sqrt{20-16\cos\theta}}$$

Now determine $\cos\phi$.

$$\sqrt{1-\cos^2\phi}=\frac{2\sin\theta}{\sqrt{20-16\cos\theta}}$$

$$1-\cos^2\phi=\frac{4\sin^2\theta}{20-16\cos\theta}$$

$$\cos^2\phi=1-\frac{4\sin^2\theta}{20-16\cos\theta}$$

$$\cos\phi=\sqrt{\frac{20-16\cos\theta-4\sin^2\theta}{20-16\cos\theta}}$$

As a result, equations (1) and (2) become

$$15\left(\sqrt{20-16\cos\theta}-2\right)\sqrt{\frac{20-16\cos\theta-4\sin^2\theta}{20-16\cos\theta}}-T_{AB}\cos\theta=0 \tag{3}$$

$$15\left(\sqrt{20-16\cos\theta}-2\right)\frac{2\sin\theta}{\sqrt{20-16\cos\theta}}+T_{AB}\sin\theta-10=0. \tag{4}$$

Solve equation (3) for T_{AB}

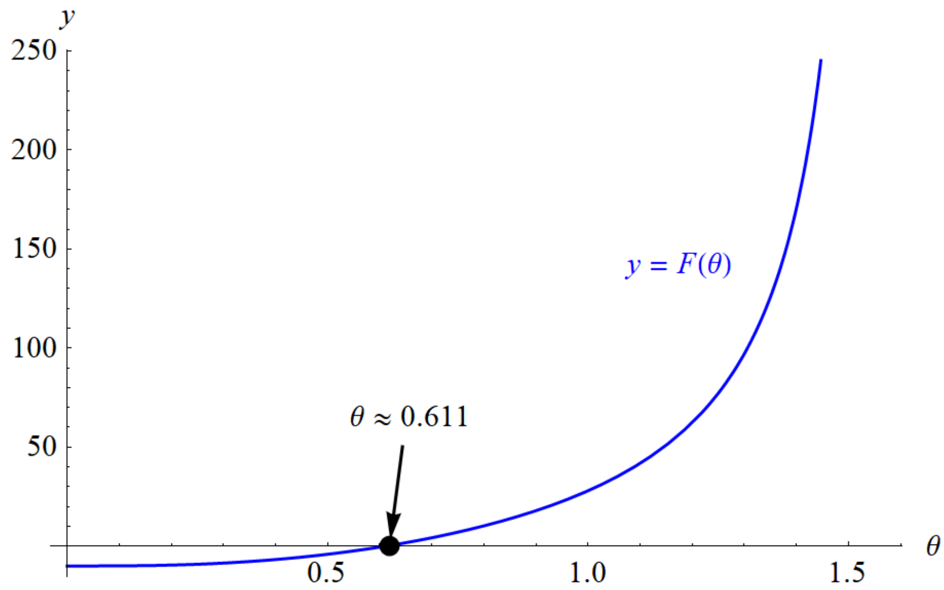
$$T_{AB}=\frac{15\left(\sqrt{20-16\cos\theta}-2\right)}{\cos\theta}\sqrt{\frac{20-16\cos\theta-4\sin^2\theta}{20-16\cos\theta}}$$

and plug it into equation (4).

$$15\left(\sqrt{20-16\cos\theta}-2\right)\frac{2\sin\theta}{\sqrt{20-16\cos\theta}}+\frac{15\left(\sqrt{20-16\cos\theta}-2\right)}{\cos\theta}\sqrt{\frac{20-16\cos\theta-4\sin^2\theta}{20-16\cos\theta}}\sin\theta-10=0$$

Let the function on the left side be $F(\theta)$.

Plot $F(\theta)$ versus θ and find where the curve crosses the horizontal axis. Since $0° < \theta < 90°$, θ only goes up to $\pi/2 \approx 1.57$.



Therefore, changing 0.611 to degrees,

$$\theta \approx 35.0°.$$